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RECENT ADVANCES IN
NUMERICAL WEATHER PREDICTION

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Recent developments in numerical weather prediction during the past several years are briefly summarized.

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1. Introduction

This review is intended to summarize the developments in numerical weather prediction which have occurred during approximately the last three years. This roughly covers the period since the completion of two major texts by Haltiner (1971) and Phillips (1973). Earlier research will be referenced only when required for the development of a particular topic. An indication of the intense activity in this field can be seen in the papers presented at the Second American Meteorological Society Conference on Numerical Prediction in Monterey, California during 1-4 October 1973, for which the abstracts have been published in the July, 1973 Bulletin of the American Meteorological Society.

Since the first application of numerical integration techniques to large scale weather prediction [Charney, Fjortoft, and von Neumann (1950)], these techniques have been applied to a large variety of atmospheric and oceanic prediction problems. These include studies of the general circulation of the atmosphere and the oceans, tropical storms, fronts, sea breezes, cumulus convection, atmospheric pollution, the planetary boundary layer and turbulence. This review will focus on the developments in large scale weather prediction, although many of the results have applications to other problems. The parameterization of smaller scale phenomena will not be considered in this review because it deserves a separate comprehensive treatment.

Numerical integration of the atmospheric equations as an initial value problem is the primary basis for the prediction of synoptic-scale disturbances for periods between 12 hours and perhaps five days and, in addition, to some extent for smaller scales and much longer periods. The sources of error in such predictions are a consequence of (a) gaps and errors in the data which make up the initial state, (b) limitations in the objective analysis-initialization schemes which are applied to the data, (c) truncation errors in numerical integration schemes, (d) incomplete representation of the many complicated dynamical processes at work in the atmosphere and finally, limitations imposed by the predictability of the atmosphere.

2. Objective Analysis and Initialization

A standard method of objective analysis which has been in use for nearly 15 years or so begins with an initial guess based on a prognosis of the mass field from the previous analysis or, perhaps simply a persistence forecast. The initial gridpoint values are then modified with actual observations weighted according to the distance between the observation and the gridpoint. Winds are obtained geostrophically or by means of a balance equation. With some care to insure vertical consistency and to avoid superadiabatic lapse rates, this type of objective analysis is quite adequate for initializing the vorticity (filtered) type of prediction models. However, the primitive equation or P.E. models introduced operationally about a half dozen years ago, are much more sensitive to the initial conditions than are the filtered models from which gravity waves have been eliminated. As a consequence, the lack of proper balance between the pressure and wind fields can give rise to quite large, spurious inertial-gravity waves with periods ranging from a few time increments Δt to a day or more. Through the natural geostrophic adjustment process, which is inherent in the P.E. models, these spurious gravity modes are eventually dispersed and suppressed to acceptable magnitudes, perhaps comparable to those observed in nature. To avoid the large pressure fluctuations associated with these spurious gravity waves, greater care should be taken to bring the objective analysis to a state where the wind and pressure fields are more nearly in proper balance before beginning the forecast, a procedure which is usually referred to as initialization.

The first efforts in this direction simply followed the practice in filtered models by using winds from a stream function obtained by solution of the balance equation with the objectively-analyzed geopotential fields on constant pressure surfaces. Since the balance equation allows for curved motion to some extent, the resulting winds are in better balance with the pressure force than geostrophic winds. Nevertheless, as shown by Phillips (1960), an appropriate divergent wind component is necessary if the inertial-gravity waves are to be excluded or negligible. Consequently, the rotational winds obtained by solution of the conventional balance equation are not sufficient to prevent the unwanted gravity modes in a primitive equation model, although they are clearly better than geostrophic winds or unmodified observed winds.

Recently, Sundqvist (1973), on the basis of limited tests, suggested that the solution of a balance equation with zero mass divergence ($\nabla_\sigma \cdot \pi \mathbb{V} = 0$) on sigma surfaces would lead to initial winds which give less gravitational noise than the winds from the conventional balance equation on pressure surfaces followed by interpolation to sigma surfaces.

In another approach, Temperaton (1973) verified that gravity oscillations were indeed present after initialization with the exact rotational part of the wind. The latter was obtained from an idealized experiment where "control" data was first generated by first running a numerical prediction model for two days using the Euler-backward scheme to dampen the high frequency modes leaving essentially only the slowly evolving meteorological modes. Then the geopotential field was treated as the observed field and a series of experiments were run for an additional day with different initialization methods. In particular the rotational wind component was extracted from the control field by solving the Poisson equation, $\nabla^2\psi = k \cdot \nabla \times V$, for the stream function ψ. Initializing with this wind resulted in gravity noise comparable to that resulting from winds obtained from solution of the balance equation.

The divergent component of the wind implied by the quasi-geostrophic theory can be obtained by solving the corresponding ω-equation and then using the continuity equation to obtain the divergent wind potential χ as follows:

$$\nabla^2\chi = - \frac{\partial\omega}{\partial p} \tag{1}$$

This is a rather lengthy task however, and is probably less effective and maybe as time consuming as simply using the model equations to achieve the desired balance. In fact, Houghton, Baumhefner and Washington (1971) made a study of initialization for the NCAR global model and found that inclusion of the vertical velocity from an ω-equation and the corresponding divergent wind component in the initial winds did not significantly reduce the trauma of inertial gravity oscillations.

At the National Meteorological Center (NMC) solution of the balance equation was abandoned several years ago in favor of extracting the rotational wind component directly from objectively analyzed winds. In addition, the 12-hour forecast divergent wind component is utilized. The resulting total initial wind field gives better agreement with the observations without significantly increasing the noise level in the prediction model.

Although they are usually treated as separate steps, objective analysis and initialization have the same goal, namely, to provide an accurate, properly balanced state from which the hydrodynamical equations can be numerically integrated forward in time. Obviously, the two steps are not dynamically independent and need not be separated, which is the view taken by a number of investigators. A specific example is Sasaki's (1958) initialization by variational methods which incorporates dynamical constraints beyond the usual geostrophic wind law in the objective analysis. These constraints have included the use of a generalized wind equation, suppression of high-frequency oscillations, as well as minimizing RMS differences between observations and a first guess field.

Lewis (1972) applied Sasaki's technique to develop an operational analysis of wind and temperature for the global band 40°S to 60°N from the surface to 250 mb. Disregarding vertical advection, Lewis derived the following generalized thermal wind equation

$$\frac{\partial}{\partial t}\frac{\partial \vec{V}}{\partial \sigma} = - \frac{\partial}{\partial \sigma}(\vec{V} \cdot \nabla \vec{V}) - f\vec{k} \times \frac{\partial \vec{V}}{\partial \sigma} - R\nabla T \equiv \vec{F} \qquad (2)$$

The variational formulation calls for the minimization of the integral

$$I = \iiint [\bar{\alpha}(\vec{V} - \bar{\vec{V}})^2 + \bar{\beta}(T - \bar{T})^2 + \alpha(F_x^{\ 2} + F_y^{\ 2})] \qquad (3)$$

where $\bar{\alpha}$, $\bar{\beta}$ are precision moduli for the wind and temperature based on variances and α is a dynamical weight factor. Solution of the Euler equations associated with the above integral provide the new wind and temperature fields. At the Monterey NWP meeting in October 1973, Sasaki applied a similar procedure to the problem of initialization by utilizing a dynamic constraint on the kinetic energy which reduced the gravity wave noise considerably.

The variational principle has also been used by Flattery (1967) at NMC in making objective analyses by fitting Hough functions (the eigenfunctions of Laplace's tidal equation) to the observed data by a least squares method. The results have been used in a global spectral model.

Another common method of objective analysis interpolates between observations to gridpoints by requiring the gridpoint values to satisfy a Poisson equation. Starting with an initial guess, the gridpoint values are then modified by the use of observed values which are treated as internal boundary points. This scheme was evaluated by Leary and Thompson (1973) for accuracy with known spherical harmonics. Wavenumber 2 was reasonably well represented with 87% of its amplitude squared appearing in the analysis field, while only 13% of the input amplitude squared of wavenumber 12 survived the analysis. Aside from demonstrating a rather severe deficiency of this analysis scheme, the study suggested that there is a less steep dropoff in the kinetic energy of the wind field, perhaps a "-2" rather than the "-3" power law obtained from spectra of such objectively analyzed data.

Several other approaches have been utilized to suppress the inertial-gravity oscillations usually present in the early stages of a P.E. forecast. One procedure consists of integrating forward and backward about the initial time starting with the objectively analyzed data and periodically restoring either the mass or the wind field at the initial time, usually the mass field in middle latitudes and perhaps the wind field in tropical latitudes. For this purpose it would appear advantageous to use an integration method which selectively damps high-frequency oscillations such as the Euler-backward scheme introduced by Matsuno (1966). As applied to a simple advective equation

$$\frac{\partial F}{\partial t} + C\,\frac{\partial F}{\partial x} = 0 \qquad (4)$$

The Euler backward scheme will damp short waves about 10% with each time step, while long meteorological type waves will be negligibly damped. However, internal gravity waves of low frequency may also dampen rather slowly. Thus considerable computing time may be required by this two-step scheme to accomplish the desired goal of suppressing the spurious gravitational noise, perhaps the equivalent of a 24- or 48-hour forecast which is operationally undesirable.

The presence of the gravity modes is usually reflected in the horizontal divergence, the vertical velocity and the surface pressure tendency. Recognizing that the divergent part of the wind is intimately related to the propagation of the gravity oscillations, Talagrand (1972) suggested a special viscosity term to dampen the divergent wind component as follows:

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,. - \mu\,\frac{\partial D}{\partial x} + \nu\,\frac{\partial \zeta}{\partial y} = 0 \\ &\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,. - \mu\,\frac{\partial D}{\partial y} - \nu\,\frac{\partial \zeta}{\partial x} = 0 \end{aligned} \qquad (5)$$

Here D is the horizontal velocity divergence and ζ, the relative vorticity. Now when the divergence and vorticity equations are formed, one obtains

$$\begin{aligned} &\frac{\partial D}{\partial t} + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,. - \mu\,\nabla^2 D = 0 \\ &\frac{\partial \zeta}{\partial t} + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,. - \nu\,\nabla^2 \zeta = 0 \end{aligned} \qquad (6)$$

It is clear that with $\nu = 0$ and $\mu \neq 0$, divergence but not vorticity will be suppressed. McPherson (1973) of NMC evaluated this novel concept with an 8-layer global primitive equation model using Shuman's semi-momentum differencing. With $\mu = 0$, oscillations primarily in the form of external gravity modes appeared in middle latitudes as follows:

AMPLITUDE	PERIOD
5 - 10 meters	3 - 4 hours
20 - 25 meters	6 - 7 hours
20 - 60 meters	10 hours

These waves were largely eliminated with a viscosity coefficient of $\mu = 2.5 \times 10^{8}\ m^{2}/sec$ in about 7 hours time. Several unfortunate side effects occurred;however, there was abnormally high surface pressure at the end of 12 hours in the vicinity of mountains together with a downstream trough. Also values of μ in excess of $10^{7}\ m^{2}/sec$ eliminated all precipitation. Clearly further evaluation is necessary. McPherson speculated that the slope of the sigma surfaces in the vicinity of mountains may be responsible. We would not find this surprising since two-dimensional diffusion on steep-sloping sigma surfaces near mountains may involve large vertical wind shear and temperature differencing, with undesirable consequences. Perhaps the diffusion technique would be more successful if carried out on quasi-horizontal surfaces, or in such a way as to avoid changing the selective vertical diffusion.

At this point we would like to return to Temperaton's experiment on dynamical initialization. He used the so-called "shallow water" equations in flux form with spherical coordinates. In previous experiments several years ago Nitta and Hovermale (1967) had initialized by integrating forward N time steps, backward 2N steps, and then returning to the initial time with N steps forward; however, N was taken to be just 1. The Euler-backward scheme gave somewhat faster convergence than the leapfrog scheme. Mesinger (1972) recommended a larger amplitude $N\Delta t$ and partial restoration of mass more frequently. Winninghoff (1971) integrated a barotropic model 18 hours forward in time and back to the initial time with the 2-step Euler-backward scheme and obtained convergence to quite good initializing winds, but obviously at great expense in computer time (equivalent to a 72-hr forecast with the leapfrog scheme). Neither friction nor heating terms were present during initialization. Friction and simulated heating after initialization gave somewhat poorer results than in the frictionless, adiabatic case. However, including these effects in the initialization procedure could lead to undesirable results depending on the mathematical formulation of these processes, which may not be reversible.

Temperaton tried restoring mass only partially at the central time but was not successful. Using a time amplitude of $4\Delta t$ and 2/3 restoration of the mass field at every other time step gave quite rapid convergence with respect to the total RMS wind error. However, gravity wave activity, as measured by the RMS divergence, was not diminished. Analysis of the winds showed that the rotational wind component converged rapidly toward the correct value but the divergent wind component was more in error than with the case N = 1. He then tried another approach of integrating forward N time steps from the initial time, then N steps backwards from the initial time and finally averaging the end results as follows:

$$u(t_o) = \frac{1}{2}[u(t_o + N\Delta t) + u(t_o - N\Delta t)] \tag{7}$$

The mass is now restored and the procedure repeated as many times as is needed to bring about convergence. The leapfrog method was used except for a first forward time step. The averaging tends to remove high-frequency modes but leaves low-frequency waves relatively unaffected. For example, the $2N\Delta t$ frequency would be removed by such averaging while the $4N\Delta t$ wave would be reduced by about one-half. Iteration cycles of $3\Delta t$, $6\Delta t$ and $9\Delta t$ were tried with the best results obtained from the $6\Delta t$ (1 hour) case. The Euler-backward method was tried for starting in each direction but showed no improvement over a simple forward step. The principal result was that considerably faster convergence was achieved using the averaging technique than with the Euler backward scheme $N = 1$. The high frequency oscillations in the RMS divergence were markedly reduced to about one-tenth the amplitude occurring with the use of the exact rotational wind component without the averaging scheme. Nevertheless, there remained an easily recognizable oscillation with a period of about four hours. Elimination of this oscillation and further reduction of the noise was achieved by first adjusting the wind field while restoring the mass field at the initializing time and then repeating the forward and backward averaging procedure except that the mass field was averaged while the newly computed winds were restored after each cycle; thus both mass and winds were adjusted. In all, 20 cycles, equivalent to a 40-hour forecast, were completed, which is again too long for operational forecasting.

Suppression of considerable gravity noise has been achieved at NMC by using a running time average of the dependent variables in connection with the leapfrog scheme due to A. Robert (1966). Specifically, when a new value of a predicted variable, say $A(t + \Delta t)$, has been calculated using the leapfrog time differencing, a weighted average of the last three time values is calculated as follows:

$$A^*(t) = A(t) + 0.5\nu[A^*(t-\Delta t) - 2A(t) + A(t+\Delta t)] \qquad (8)$$

The new value $A^*(t)$ is now used in the next leapfrog step to $A(t + 2\Delta t)$, that is

$$A(t + 2\Delta t) = A^*(t) + 2\Delta t \left(\frac{\partial A}{\partial t}\right)_{t+\Delta t} \qquad (9)$$

The weight factor ν controls the amount of smoothing; $\nu = 0.5$ gives the familiar 1-2-1 averaging. The simpler averaging function with $A(t - \Delta t)$ instead of $A^*(t-\Delta t)$ in Eq. (8) would completely remove waves of period $2\Delta t$ and dampen a $4\Delta t$ wave by about one half, while much longer periods are relatively unaffected. Recall here that the spurious computational mode generated in the leapfrog scheme has very nearly a period of $2\Delta t$ if a corresponding physical mode is of a relatively lower frequency. As a consequence, NMC's averaging procedure is effective in removing the computational mode. Moreover, repetition of the averaging at every time step tends to damp somewhat lower frequencies, including a considerable part of the spurious gravity noise generated through the early hours of a P.E. forecast. Asselin (1972) has computed the damping and phase shift characteristics as a function of frequency for a time filter of this form. Low frequency meteorological waves show negligible amplitude change and phase shift, but there is very effective damping of computational modes.

In some recent initialization experiments with a five-level, global P.E. model, Haltiner and McCullough (1974) did not find any significant reduction of gravity noise with initial rotational winds from a balance equation solved on σ-surfaces as compared to similar winds from a p-surface balance equation. This may be somewhat surprising since from theoretical considerations, it might be expected that at least external gravity waves would be suppressed. Sundqvist assumed zero mass divergence, $\nabla_\sigma \cdot \pi \vec{V} = 0$, where π is surface pressure, to obtain the balance equation on sigma surfaces. It follows from the integrated continuity equation, namely, $\partial\pi/\partial t = -\int_0^1 \nabla \cdot (\pi v \vec{V})\, d\sigma$, that if $\nabla \cdot \pi \vec{V} \equiv 0$, the surface pressure tendency will vanish initially which should tend to suppress external gravity waves. In any event, the Haltiner-McCullough initialization experiments with the σ-balanced winds did not show significant improvement in damping spurious inertial-gravity waves.

The Robert time filter (1966) is not only effective in eliminating the computational mode associated with the leapfrog scheme, but also gradually tends to dampen frequencies characteristic of the gravitational noise at the beginning of a P.E. forecast. Haltiner and McCullough combined the time filter with the averaging scheme of Temperaton to substantially reduce the noise in the equivalent computer time of a 12-hr forecast. The latter was accomplished by twice averaging winds from a 3-hr forecast and a 3-hr hindcast and restoring the mass field to the initial values for each time. The winds thus obtained together with the original mass field constituted the initial conditions for prediction, which resulted in a substantial decrease in high frequency surface pressure oscillations compared to the immediate use of the balanced winds for initialization.

The addition of the divergent wind component predicted by the model from a previous analysis to the present analysis time should further aid in damping the spurious inertial-gravity noise generated by the introduction of new data. This step costs very little in computer time.

3. Data Assimilation

The advent of the satellite infrared spectrometers (e.g., SIRS), which can sound the atmosphere from space, has provided a new and important source of data which is especially valuable over sparse-data areas. It did, however, further emphasize a problem of how to utilize most effectively upper air data which do not occur at or very near the regular synoptic observation times, a problem already existing in connection with aircraft observations. The procedure for incorporating off-time reports into an analysis-prediction scheme is referred to as <u>four-dimensional assimilation</u>. Clearly the trauma of spurious inertial gravity waves associated with initialization must be eliminated if new observational data is to be more or less continuously incorporated into a prediction procedure.

Some of the early idealized experiments dealing with the assimilation of SIRS data by Charney, et al (1969), Jastro and Halem (1970) and Williamson and Kasahara (1971) showed that the simple insertion of temperature data can eventually determine the wind field and vice versa. The process of inserting some variables into a numerical forecast while others are unchanged has been referred to as <u>updating</u>, which seems to be a rather restrictive definition in terms of operational forecasting. In any event, the degree to which one field, for example, winds, can be obtained by updating another, perhaps temperature, depends on the predictability of the model which, in turn, depends on natural instabilities, nonlinear exchanges, frictional dissipation, etc.. Using model data to stimulate the real world, then introducing errors and updating with, quote, "observed", data provides an indication of the best possible results that could be obtained by updating an operational prediction model with real data. In actual operational forecasts there are other sources of errors including the representation of the physical processes and numerical truncation. Naturally, the better the model, the less the predictability error, but simple insertion of data is assuredly not the best way to update. It would be quite logical to modify other variables by static or dynamic balancing and refer to the whole procedure as updating or four-dimensional data assimilation.

Mesinger (1972) suggested the following three names for methods of obtaining a proper balance between the mass and motion fields:

(1) Static Balancing - use of a wind law such as the geostrophic relation or the balance equation.

(2) Dynamic Balancing - going backward and forward about a central time, restoring mass or wind at $t = 0$ or letting both vary. Sasaki's variational method can also be considered as a combination of static and dynamic balancing.

(3) Four-Dimensional Balancing in space and time - introduction of data three-dimensionally periodically at discrete time intervals, including regular synoptic times.

Hayden (1972) used a 2-layer P.E. model with Shuman's semi-momentum differencing scheme to run a series of experiments with the insertion of temperature calculated from geopotentials which were in turn derived from the SIRS temperatures. Data were inserted six times during a 12-hr forecast. Temperature tendencies over a period of one orbit were generally less than the observational error; consequently, as found by Talagrand (1971), too frequent updating may be deleterious. Hayden found that a static balancing, consisting of a geostrophic wind correction, ΔV, computed from the changes in geopotential $\Delta\phi$ inferred from the temperature insertions aided the geostrophic adjustment process and reduced the shock of updating. He also attempted dynamic balancing with $N = 2$, i.e., integrating two steps forward, $4\Delta t$ backward and $2\Delta t$ forward to return to the initial time.

Three criteria were used to measure the successful assimilation of data: (a) the updating must not unduly shock the model which he inferred from a measure of the divergent wind component; (b) does the model remember the data inserted (this was tested by reversing the forecast after 12 hours and noting whether the hindcast temperatures were nearer to the inserted values than were the original temperatures that existed prior to the updating.); (c) lastly does the updating result in better mass and wind distributions when compared to the NWS analyzed fields? For this purpose the 12-hour forecast was followed by a 12-hour hindcast. Then the difference between the cycled data and the initial data were correlated with the difference between the NWS analysis and the initial state; a positive correlation indicates success.

The following conclusions were reached:

a. Even with poor initial conditions, temperature can be assimilated without shock if some balancing is performed to aid the geostrophic adjustment process. Dynamic balancing is sufficient but static balancing in the form of a simple geostrophic wind correction speeds up the assimilation process considerably, at least outside the tropics.

b. Hayden anticipates that under operational conditions, where the model state is maintained similar to the observed state, four-dimensional assimilation can be accomplished without time-consuming dynamic initialization.

c. Four-dimensional data assimilation is evidently more effective than regular objective analysis of off time reports that have been updated to regular synoptic times by Lagrangian advection. The exact details of the surface pressure appear to be relatively unimportant to the effectiveness of the four-dimensional assimilation. However, the SIRS-B data by themselves are not capable of defining the circulation because the data density is not sufficient for objective analysis, nor with the model, of producing even an approximate surface pressure field.

Bengtsson and Gustavsson (1971) had previously found also that analysis prior to insertion of data, say from a satellite, leads to a more rapid reduction of error. Talagrand and Miyakoda (1971) showed that a synthesis technique of averaging forecasts made from objective analyses made at different times can reduce the random errors of measurement and analysis, sort of a Monte Carlo approach. They did some studies of inserting data into a running forecast when and where the data were available. If the difference between the predicted values and the observations is less than or equal to the observational error, don't insert it; there's no point to shocking the system and uselessly creating spurious inertial-gravity waves.

In summary, experimentation thus far suggests that one or several of the following steps are useful during initialization and four-dimensional data assimilation.

Firstly, at the regular 0000Z and 1200Z synoptic times:

a. Perform a preliminary objective analysis and obtain a stream function by solution of the balance equation, perhaps on the surfaces of the vertical coordinate, or as an alternative obtain the rotational wind directly from objectively analyzed winds. A combination of both may be desirable, the former in middle latitudes and the latter in the tropics.

In the tropics where the observational errors in the pressure field may be comparable to the pressure variations associated with synoptic disturbances (except for tropical storms), it is generally unwise to attempt to obtain the stream function from the pressure field by solving the balance equation, which, in fact, is singular at the equator. A recommended procedure is to calculate the vorticity directly from the observed <u>wind field</u> and then obtain the stream function by solving the Poisson equation $\nabla^2\psi = \zeta$. Finally, the geopotential field is calculated from the stream function with some form of the balance equation. Saha and Suryanarayana (1971) made a series of calculations of the geopotential in this manner from the quasi-geostrophic relation, the linear balance equation, the balance equation and the so-called vorticity form of the balance equation which are, respectively,

$$
\begin{aligned}
\nabla^2\phi &= f\nabla^2\psi \\
\nabla^2\phi &= \nabla \cdot (f\nabla\psi) \\
\nabla^2\phi &= 2J\left(\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}\right) + \nabla \cdot (f\nabla\psi) \\
\nabla^2\phi &= \eta\zeta + k \cdot \nabla\eta \times \vec{V} - \nabla^2(V^2/2)
\end{aligned}
\tag{10}
$$

Observed winds were used to evaluate the vorticity, $\zeta = k \cdot \nabla x V$, and ψ was obtained from $\nabla^2\psi = J$. The geopotential fields obtained from the last three forms were very similar and compared favorably with the analyzed geopotential fields at the 850, 700, 500 and 300-mb levels. However, the last equation, the vorticity form, gave the least RMS error.

b. To supplement the rotational wind component, the divergent wind component predicted from a previous analysis may be used. The cost in computer time amounts to the solution of a Poisson equation, $\nabla^2\chi = \nabla \cdot V$, followed by the calculation $\vec{V} = \nabla\chi$.

c. Next apply some dynamic balancing such as Temperaton's technique.

Step a and perhaps b may be replaced by a variational method of analysis including some dynamical constraints.

Between synoptic times, insert new data where available at intervals of not less than three hours with an objective analysis in a limited region of the reports including first static balancing to adjust the wind or the mass fields partially to one other. This may be followed by dynamic balancing to reduce further the gravitational noise if the differencing scheme and frictional terms are insufficient for this purpose.

4. Integration Methods

a. Semi-implicit schemes

Although the momentum or primitive equations are simpler and involve fewer approximations than the filtered equations, the presence of gravity waves requires a much smaller time step to avoid computational instability with explicit integration schemes. Otherwise the small step is of little advantage or even harmful insofar as meteorological waves are concerned, and it is certainly expensive in terms of computer time. As a consequence, there has been considerable effort to circumvent the stability requirement. In the Soviet Union, Marchuk (1965) introduced a differencing scheme which treated the gravity modes implicitly and the low frequency meteorological modes explicitly, thus permitting a much larger time step. Kwizak and Robert (1971) successfully applied a semi-implicit differencing method similar to one suggested by Kurihara (1965) to a barotropic 500-mb forecast. To illustrate the technique we adopt the notation of Elvins and Sundstrom (1973) applied to the shallow water equations, which in differential form are

$$u_t = - \phi_x - u\,u_x - v\,u_y + fv$$
$$v_t = - \phi_y - u\,v_x - v\,v_y - fu \qquad (11)$$
$$\phi_t = - \phi\,(u_x + v_y) - (u\phi_x + v\phi_y)$$

The pressure gradient terms in the momentum equations and the divergence term in the continuity equation, which together primarily govern the propagation of gravity waves, are evaluated semi-implicitly while the remaining terms are evaluated explicitly.

In the difference equations u^{n+1} and v^{n+1} in the continuity equation are replaced by substitution from two momentum equations obtaining an elliptic equation in ϕ^{n+1}.

$$[1-\Delta t^2\,\phi^n\,(D_{ox}^{\ 2} + D_{ox}^{\ 2})]\,(\phi^{n+1} - \phi^{n-1}) = F(x,y,t_n,t_{n-1}) \qquad (12)$$

Here F is composed of terms at times t_n and t_{n-1}. After solving this equation for ϕ^{n+1} by one of the usual methods, e.g., relaxation, u^{n+1} and v^{n+1} may be calculated directly from the momentum equations. As a consequence of the implicit character of the difference equations with respect to gravity waves, a much larger time step is permitted without encountering linear computational instability. Of course, extra time is taken at each step to solve the elliptic equation, but overall, computer time is reduced by a factor of 3 to 4.

Elvius and Sundstrom (1973) suggested an efficient differencing system which is staggered in both space and time. This scheme not only permits a larger time step but also reduces the phase speed error of the low frequency or "meteorological" mode. They also developed suitable boundary conditions for use with a fine mesh model, which is undergoing further tests with realistic initial conditions. Treating the coriolis terms implicitly permits a slightly larger time step.

Baroclinic models are more complicated, but the semi-implicit technique is applicable in a similar manner. Gerrity, McPherson and Scolnik (1973) have developed the semi-implicit difference equations using Shuman's semi-momentum differencing technique for the NMC 6-layer primitive equation model. The model has run stably for four days; however, it is not yet operational.

When the implicit scheme is used, the phase velocities of gravitational oscillations are greatly reduced, hence the geostrophic adjustment process can be retarded, as will any initialization procedure for damping spurious inertial-gravity waves. Experiments by McPherson and Kistler (1973) verified the delayed damping of the gravity waves; however, because of the larger time step, dynamic initialization can be accomplished with less computer time. The net result was still a gain of 2 to 1 over the explicit integration during an initialization procedure.

5. Direct Methods for Helmholtz and Poisson Equations

The need to solve Helmholtz-type equations in connection with the semi-implicit methods has stimulated interest in the more recent "direct" methods of solving Poisson and Helmholtz equations. Leslie and McAvaney (1973) have compared the speed and accuracy of a number of methods for solving equations of the form:

$$\nabla^2 \phi - \alpha(x,y)\, \phi = f(x,y) \tag{13}$$

where the Helmholtz coefficient α and the forcing function f are known.

When approximated by the usual finite difference analogues, the foregoing equation is expressible as a system of linear equations with a block tridiagonal matrix of coefficients. In the past, iterative methods such as the Liebmann successive over-relaxation method (SOR) have been widely used because of their simplicity. However, faster direct methods developed in recent years are replacing the relaxation methods, at least when a rectangular region is involved with simple boundary conditions. The direct methods may be divided into four categories as follows:

a. Block Method which uses the fact that A is block tridiagonal.

b. Cyclic Reduction Method (DCR) which reduces the dimensions of the matrix to be solved in a recursive manner. This method is restricted to certain numbers of interior points, such that with an MXN grid either M or N must equal $2^k - 1$.

c. Matrix Reduction which through coordinate transformation reduces the problem to a simpler tridiagonal form that has an easily computed solution. The dimension reduction method (DRM) is relatively simple when a fast Fourier transform is available.

d. Finally, in certain circumstances the fast Fourier transform can be applied directly.

Table 1 shows some comparative solution times and accuracies (RE) of the DCR, DRM and SOR methods applied to Poisson and Helmholtz equations on a 65 x 65 grid.

TABLE 1

(from Leslie and McAvaney)

	DCR		DRM		SOR	
	RE	TIME	RE	TIME	RE	TIME
	10^{-6}	0.89	10^{-6}	1.25	10^{-6}	42.3
POISSON	10^{-13}	0.90	10^{-13}	1.27	10^{-10}	75.9
		5.3		7.5	10^{-2}	6.4
					10^{-4}	11.0
HELMHOLTZ					10^{-6}	16.3

It is clear that with respect to Poisson type equations, the direct methods are far superior. To solve the Helmholtz equations by direct methods, the Helmholtz coefficient has to be a constant. On the other hand, the iterative (SOR) methods permit a variable Helmholtz coefficient and give rapid convergence when that coefficient is large. This technique can then compare favorably with the direct methods, particularly if a little less accuracy is acceptable, say, RE $\sim 10^{-4}$. Also the SOR methods, which are most readily generalized to irregular domains and mixed boundary conditions, are simple to program and require minimal storage in comparison to direct methods. But it is hoped that the lack of flexibility in the direct methods will be overcome in view of the importance of solving Helmholtz equations in connection with semi-implicit methods which can be real time savers with respect to computing.

6. Global Grids

As computing capability has improved in the last several years efforts have increased toward operational global forecasting and also in fine mesh models. The absence of lateral boundaries in a global model is an important advantage since such unnatural barriers cause errors in hemispheric models that eventually propagate from the fictitious boundaries in the tropics into middle latitudes. Numerical forecasting in the tropics, which admittedly has severe limitations at present, would be quite hopeless if artificial boundary conditions are imposed in low latitudes. On the other hand, a tropical band with middle latitude walls is not satisfactory either because these boundaries are far too active. So the only alternative appears to be global models despite the vast areas of little or no conventional data in the Southern Hemisphere, albeit the weather satellites are helping to overcome the data problem.

A significant difficulty with global models is the lack of a suitable plane projection which does not seriously distort some areas. The most natural approach is a latitude-longitude grid; however, the convergence of the meridians poses a knotty problem, for as the distance between equal longitude spacing shrinks toward the poles, a shorter time step is needed to maintain linear computational stability. A time step short enough to maintain stability in polar regions is exceedingly wasteful in low latitudes. Various techniques have been used to overcome this difficulty with varying degrees of success. The most common approach of late has been to filter out or dampen the short waves that would lead to instability near the poles so that a relatively large time step can be used throughout, rather than simply decreasing the time step with increasing latitude. In the Arakawa (1972) - Mintz model, the procedure consists of temporarily Fourier analyzing fields which will be differenced with respect to longitude and then modifying such Fourier amplitude so that the CFL stability criterion can be satisfied without shortening the time step. This principle can be illustrated with a simple advection equation for which the stability criterion for wave number k and maximum wave speed c is typically of the form

$$\frac{c\Delta t}{d_j} \sin kd \leq \epsilon \tag{14}$$

where $d_j = a \cos \varphi_j \Delta\lambda$ is the grid distance at latitude φ_j and ϵ is a constant perhaps unity. By reducing the amplitude of each wave component of the longitudinal gradient at each latitude by a factor S_{jk} the criterion becomes

$$(S_{jk}) \frac{c\Delta t}{a\Delta\lambda \cos \varphi_j} \sin kd \leq \epsilon \tag{15}$$

It is clear that for a fixed $\Delta\lambda$ and Δt, computational stability can be maintained by decreasing S_{jk} as the latitude and wave number increase. This type of procedure is applied to all longitudinal derivatives in the terms involving gravity wave propagation.

Vanderman (1970) used running averages of the tendencies to filter high frequencies components; however, this can cause computational instability. At the NOAA Geophysical Fluid Dynamics Laboratory Holloway, Spelman and Manabe (1973) applied space filtering to all time integrated variables at each time step. The filtering limits the east-west wavelength at all latitudes to the distance of two gridlengths at the equator. The minimum wavelength is given by $L_{min} = 4\pi a N^{-1}$, where N is the number of gridpoints around a latitude circle and a is the radius of the earth. The maximum number of waves at latitude θ is

$$\frac{2\pi a \cos \theta}{L_{min}} = (N/2) \cos \theta \tag{16}$$

which is rounded to the nearest integer. This is accomplished by a Fourier analysis followed by synthesis with only the desired waves at each latitude circle included. The procedure has no significant effect on quadratic conservation properties of the differencing schemes. The components of vector variables are first transformed into polar stereographic coordinates to avoid the problems associated with averaging vectors from widely separated longitudes where the unit vectors differ substantially in direction as discussed by Shuman (1970).

When tested on barotropic and baroclinic models the foregoing procedure proved to be superior to the Kurihara grid which has a poleward decrease in the number of gridpoints per latitude circle in such a way as to keep the distance between gridpoints from decreasing appreciably. The problem of spurious anticyclogenesis at the poles associated with a Kurihara grid was diminished. It was concluded that fast Fourier methods and new parallel computers can provide the necessary speed to handle the extra gridpoints in polar regions when the simpler differencing schemes are used with spherical coordinates.

Some tests were also conducted by Williamson and Browning (1973) with global grids which verified this conclusion. They found with a grid that is uniform in a curvilinear coordinate system, the accuracy of approximations involving curvilinear velocities is less near the singularities. In order to avoid the small time step associated with a uniform grid, they tried the method of skipping points near the poles to maintain a more nearly uniform distance between gridpoints, but the skipped grid resulted in large errors. More accurate interpolations did not help this matter. However, by applying the Fourier technique to remove short wavelengths the errors were comparable to a uniform grid requiring a much shorter time step.

7. Fourth-Order Differencing

In another aspect of computation-time vs accuracy, it has been shown that a greater reduction in phase error can be achieved per unit of extra computer time by selective use of fourth-order space differencing than by reducing the grid size [see, for example, Williams (1972)]. This may be illustrated with the simple equation (4). Suppose Δt is chosen to maintain linear computational stability in a P.E. model which permits a fast external gravity wave with a phase speed C_g of perhaps 300 m/sec or more and $C_g \Delta t/\Delta x \simeq 1$. For the slower meteorological waves, with phase speed, say C_m, the ratio $C_m \Delta t/\Delta x$ will be much less than one, perhaps a tenth, or so. Some computations were made for a centered difference form of (4) with both second- and fourth-order space differences and second-order time differencing (leapfrog). As an illustration of the improvement in phase speed accuracy, assume $C_m \Delta t/\Delta x = 0.2$. Then the ratios of the finite difference phase speeds to the true speed for several wavelengths are as follows:

L	$4\Delta x$	$6\Delta x$	$8\Delta x$	$10\Delta x$	$12\Delta x$
2nd order	0.64	0.84	0.91	0.94	0.96
4th order	0.86	0.97	0.99	1.00	1.00

Although this is a much simplified illustration, improvements in phase speed accuracy of 5 to 20% or more can be expected with fourth-order space differences. Moreover, the latter need only be applied to the terms governing the propagation of meteorological waves and not the terms involving gravity wave propagation. It should be mentioned that the fourth-order approximation required a somewhat smaller time step, perhaps 20 to 25%, to maintain linear computational stability [see Haltiner and Williams (1973)]. On the other hand, halving the grid distance in the simple one-dimensional model described above would quadruple the computation time, yet the resulting improvement in phase speeds would be roughly the same as going from second to fourth order, as may be seen by comparing the $4\Delta x$ and $8\Delta x$ ratios above (0.64 and 0.91), or the $6\Delta x$ and $12\Delta x$ values, 0.84 and 0.96.

8. Mesh Models

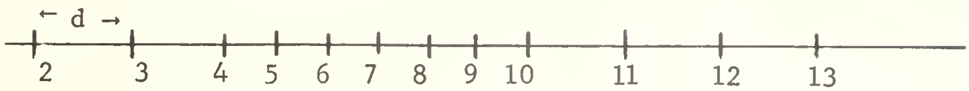
Because of the enormous range of scales of atmospheric phenomena and limitations of even the most modern computers, it is obviously impossible to model numerically all phenomena on a single mesh of uniformly spaced gridpoints. Scales which are too small to be represented with a specified length must be parameterized in terms of the large-scale variables if their influence is to be felt. On the other hand, influences from outside the region of integration can be imposed through the boundary conditions, but both procedures may be inadequate at times. It is noteworthy that some important mesoscale phenomena may be infrequent in occurrence and be predictable for only a short range of time. Consequently, it is desirable to superimpose a fine mesh grid (FMG) on a coarse mesh (CMG) covering a much larger area, perhaps a hemisphere or the entire globe. Quite a few meteorological organizations, both foreign and domestic, are currently carrying out numerical integrations on fine-mesh, limited area grids.

One of the most critical problems in dealing with limited area forecasts, including the superposition of different grids, is the treatment of the boundary conditions. This problem is not really new and had to be faced in the first numerical prediction experiments by Charney, Fjortoft and von Neumann (1950). They concluded correctly that the optimal procedure was to specify precisely as many boundary values as required by the corresponding linearized equation to have a well-behaved solution. Additional values needed for the finite difference equations should be computed by extrapolations from interior values. Apparently their method of extrapolation proved to be unstable, as later shown by Platzman (1954), and they simply specified all values on the boundary. This gave stable results which, however, were less accurate and more stringent than necessary. Having to maintain constant values along inflow boundaries can lead to large errors propagating into the forecast region; however, this is certainly less of a problem when fine-mesh boundaries are permitted to change periodically through the use of coarse-mesh forecasts. Nevertheless, the latter situation is in a sense a special case of the limited area forecast, for although the fine-mesh boundary values are no longer constant, it is necessary to obtain them by interpolation in space and time from coarse-grid values. The objective then is to do this in such a way as to avoid any instability at the boundaries and to obtain the most accurate forecast possible.

The proper procedure for a barotropic primitive equation model, proposed by Elvius and Sundstrom (1973), following Charney (1962), is to prescribe at all boundary points the quantity $V_n - \varphi/\sqrt{\bar{\varphi}_o}$, which corresponds to the outgoing characteristic. This permits gravity waves to leave the region rather than be reflected. In addition, the tangential component of velocity is prescribed at inflow points. Over-specification of boundary values is less accurate and may lead to parasitic waves and perhaps instabilities. For a baroclinic system the situation is much more complicated. Sundstrom (1973) recommends an approximation which specifies the tangential velocity at inflow points as above while at outflow points the tangential velocity is extrapolated from the interior. Next a combination of normal velocity component and potential temperature corresponding to the inward external gravity wave is specified on all boundaries, and the combination corresponding to the first outward gravity wave is computed by extrapolation from interior values in such a way as to avoid instability. This would not allow the internal gravity waves to leave the region.

Returning now to actual experiments with nesting of grids, the early efforts of superimposing a fine mesh on a coarse mesh consisted of integrating the coarse mesh model, for say 24 hours, and saving the "history tape" of data at every time step. Then the boundary values for the fine mesh model are interpolated in space and time from the CMG predictions, which is an overspecification and less accurate but nevertheless is computationally stable [Elvius and Sundstrom (1973)]. Here the FMG predictions are influenced by the large scales depicted on the CMG, but not vice versa. Nevertheless, the fine mesh gives better resolution of physical features and permits smaller scale disturbances to develop. Some examples of this approach are the experiments of Hill (1968), Wang and Halpern (1970).

A successful meteorological experiment which permitted interaction between the CMG and FMG through simultaneous integrations was carried out by Harrison and Elsberry (1972). By utilizing a movable fine mesh centered over a traveling one-dimensional gravity wave the FMG produced forecasts comparable to those obtained by a fine mesh everywhere. The boundaries of the adjacent CMG and FMG overlapped as illustrated below.



In their scheme the FMG integration determines the values at points 6 and 8; then these values are used for subsequent predictions for points 4 and 10 in the CMG. Similarly, the FMG predictions for points 5 and 9 utilize the values at points 4 and 10 determined by the CMG integrations. Thus, for example, with a simple advective equation $\partial u/\partial t + U\,\partial u/\partial x = 0$, the interface equations are

$$\frac{\partial u_4}{\partial t} = -\frac{U}{2d}(u_6 - u_3) \text{ , and} \tag{17}$$

$$\frac{\partial u_5}{\partial t} = -\frac{U}{d}(u_6 - u_4) \tag{18}$$

The computed changes for the larger time steps at the CMG points which form the boundaries of the FMG (i.e. points 4 and 10) are proportioned equally to supply the intermediate temporal values of u_4 which are needed at the smaller time intervals to integrate the adjacent FMG points (i.e. points 5 and 9). Similarly, linear spatial interpolation is performed to obtain values between CMG points as needed for the FMG integration. As a consequence of this procedure the FMG integration influenced the CMG, and very importantly the integration procedure is stable and created no significant noise or near-discontinuities at boundaries.

Applying this procedure to an idealized two-dimensional tropical disturbance and keeping the fine mesh grid centered over the disturbance gave results equivalent to a fine mesh everywhere in terms of forecast intensity and also for energy fluxes across the boundaries dividing the grids. However, some "noise" did develop at the boundary. This was suppressed by smoothing across the boundary and the addition of a small diffusion term.

Phillips and Shukla (1973) considered the two strategies of <u>one-way interaction</u> using the history tape and the <u>two-way interaction</u> such as the one just described. By a heuristic argument based on the method of characteristics, they inferred that the two-way interaction procedure would give a more faithful reproduction of the proper transmission of information into and out of the fine-mesh region. Some numerical tests with the shallow-water equations showed that the two-way strategy did indeed lead to less error. They also found that the Lax Wendroff two-step scheme gave a somewhat larger error at 12 hours than did the leapfrog scheme, but the reverse was true at 24 and 48 hours.

Ookochi (1972) combined a fine mesh with a coarse mesh for the integration of barotropic primitive equations in flux form on a staggered grid. The results were essentially a composite of complete fine-mesh and coarse-mesh integrations with no significant noise at the boundaries. The principal integral properties involving mass, total energy, etc. were well conserved during the 96-hour experiment.

Harrison (1973) describes some further experiments with systems of two and three nested grids for the simulation of a tropical storm by integration of the primitive equation on a four-level model. His calculations demonstrated the feasibility and advantages of nested grids in savings of computer time.

Presumably better phase speeds would be achieved as a consequence of the fine mesh because of smaller truncation errors. Note, however, that any wave, particularly those that are poorly represented on the coarse grid, will change phase speed when passing through the interface into the fine mesh, and again later when it leaves the fine mesh. As a consequence, erroneous interaction can occur with that part that remains in the coarse net.

9. Vertical Coordinates

The most commonly used vertical coordinate in primitive equation prediction models is the sigma coordinate (σ) which leads to $\sigma \equiv 1$ and $d\sigma/dt = 0$ on the lower boundary. This simplification of the lower boundary conditions is accompanied by a more complicated expression for the pressure gradient force.

The use of potential temperature as a vertical coordinate [Eliassen (1962)] has received new attention in the last few years. If there is no heating the isentropic surfaces will move as material surfaces. This feature is very helpful for resolving frontal zones and sharp upper level jets. A possible disadvantage of the isentropic coordinates is the fact that the potential temperature shows a large variation along the lower boundary. Primitive equation integrations have been carried out by Eliassen and Raustein (1967), Gall (1972) and Shapiro (1973) with isentropic coordinates. They encountered no major difficulties, and they obtained good simulations of the jet stream structure. Gall (1972) included latent heat in his study. Bleck (1973 a,b) made forecasts with real data with the quasi-geostrophic form of the isentropic potential vorticity equation. The forecasts showed skill in predicting cyclone development. The isentropic models show considerable promise for limited area models which treat the smaller scale synoptic features. These models have not been tested in global formulations or for longer period forecasts.

10. Forecasting Skill

With the greater sophistication in numerical modelling and the large advances in computer technology, it may be rightly asked whether there have been concomitant improvements in forecasting skill. To answer this question we may turn to verification statistics published by the National Weather Service which are probably indicative of other groups as well. Figure 1 from Shuman (1972) shows the S_1 scores, which are approximately a measure of the normalized RMS vector error of pressure gradient, for 30-hour, sea-level (upper scale) and 500-mb (lower scale) forecasts for North America from 1948 to 1971. (Shuman states that in terms of practical skill, scores of 0.30 at sea level and 0.20 at 500 mb are nearly perfect while scores of 0.80 at sea level and 0.70 at 500 mb are considered essentially worthless.) There is clearly a general downward trend over the years indicating increasing skill. The improvement in the latter half of the 50's and early 60's is ascribed to the introduction and continuing improvements in the barotropic numerical 500-mb forecasts, while at sea level there was increasing skill on the part of prognosticators in the use of the 500-mb forecasts.

The first successful baroclinic model, a three-level filtered version, became operational in 1962, which is not reflected clearly in the graphs because the years 64-66 proved to be a more difficult period to forecast and barotropic forecasts suffered as well. Nevertheless, a new plateau of skill was established and there was a general downward trend of S_1 scores. The first surface baroclinic model was a simplified graphical type due to

Dr. Richard Reed which went into operation in 1962 and drew from the independently-made numerical 500-mb predictions. The continuing improvement in surface predictions from 1962 to 1966 was largely a result of Reed's model.

The NMC 6-layer primitive equation has been under continuous development since 1966; it is the first numerical model at NMC to produce a better sea-level prediction than manual predictions without NWP guidance. However, with the numerical product in hand, man can improve the prognosis by about five points on the average in terms of S_1 scores. The net result has been a continued improvement in the skill scores.

Figure 2 from Cooley and Derovin (1972) shows the NMC average 36-hour, 500-mb S_1 scores. The light shading shows the human improvement. Figure 3 shows the 30-hour surface scores.

Considering the marked increase in accuracy of surface and 500-mb prognoses the corresponding improvement in forecasts of weather elements is somewhat disappointing. Precipitation-no precipitation forecasts for the periods 12-24 and 24-36 hours improved only about 5% between 1959 and 1970. There was somewhat greater improvement in temperature forecasts. For example, the annual number of maximum temperature forecast errors equal to $10^{\circ}F$ or greater at Salt Lake City decreased from 60 in 1949 to 22 in 1971.

Sanders (1973) recently reported on six years of forecasting temperature and precipitation by <u>staff</u> and <u>students</u> of Massachusetts Institute of Technology for the local observation site at Logan Airport. Despite continuous improvement in predicted synoptic patterns at the surface and aloft, there was no increase of skill in temperature and precipitation forecasts as measured by the incremental accuracy over the climatological mean. As may be expected, skill decreased with increasing length of forecast and more rapidly with precipitation than with temperature. Minimum skill occurred in summer, probably due to the greater influence of mesoscale phenomena. Sanders also indicated that the lack of improvement in forecasting temperature and precipitation and even a slight downward trend in the latter is also shared by NMC forecasters. He suggests that the reason for the first day forecasting difficulties may lie in the fact that the benefits of short-range synoptic-scale forecasts of the mass field have been maximized and that the source of the errors in precipitation and temperature forecasts are primarily a result of mesoscale phenomena such as fronts, land-sea circulations, convection, urban influences, etc. According to Sanders' results skill in precipitation forecasts drops to 10% in 2.5 days and in four days for temperature. He suggests that something of a breakthrough in synoptic forecasting is needed to improve significantly prediction beyond the first day.

Despite some variations in the trends of forecasting skills, it is safe to conclude that, overall, forecasting ability has shown definite improvement since the advent of numerical weather forecasts. Moreover, further improvements can be expected in the latter half of the 70's as the current research efforts in numerical techniques, simulation of the physical processes, and initialization techniques become operational, along with better satellite data and smaller mesh sizes.

We may now ask the question is there no limit to the ultimate range and accuracy of weather forecasts if one is willing to spend enough money to provide the necessary data and enough computing power for the calculations? To answer this question the predictability of the atmosphere must be considered.

11. Atmospheric Predictability

Lorenz (1965) pointed out that perfect prediction can never be expected because (a) the governing laws are not perfectly known but only approximations, (b) nor is the system of equations strictly deterministic and finally (c) the initial state is not perfectly known. Moreover, he shows that even for a determinate system of equations, such as one normally used in numerical weather prediction, separate solutions which are nearly identical at the initial time do not remain nearly identical as time progresses, but eventually become as different as two solutions chosen randomly at the same time and day of the year. The atmosphere contains some periodic oscillations - principally the annual and diurnal variations and their overtones - which are predictable at essentially infinite range. However, accurate long range prediction of the remaining features is impossible because the initial state of the atmosphere will always be imperfectly known.

Lorenz (1969a) suggested three approaches that might be used to estimate the predictability of the atmosphere:

a. The dynamical approach wherein a system of equations closely resembling the atmospheric equations is integrated from slightly different initial conditions and then the rate of amplification of the differences is determined;

b. The empirical approach which utilizes similar weather types, usually referred to as "analogues" and determines their subsequent separation in statistical terms as a function of time;

c. The empirical-dynamical approach which utilizes a new set of equations (derived from the atmospheric equations) which describes a spectral distribution of forecast errors.

Most of the studies which follow (a) used general circulation models which had been integrated until they were in approximate balance. At this time another integration was begun with an initial state which consisted of the balanced field plus a small departure. As this solution departed from the control experiment the growth rate of the error field and the range of predictability were obtained. Experiments by Mintz and Arakawa, Smagorinsky and Leith have been summarized by Charney, et al (1966). Charney estimated the RMS doubling time of small errors to be about 4 to 5 days and a predictability limit imposed by typical observational errors of about two weeks. Smagorinsky (1969) presented the results of experiments carried out at the NOAA Geophysical Fluid Dynamics Laboratory with their 10-level primitive equation general circulation model. A random temperature disturbance with a standard deviation of $0.5^{\circ}C$ at all levels was added to the control run. The vertical average of the standard deviation dropped from $0.5^{\circ}C$ to $0.2^{\circ}C$ after one day, reflecting a "geostrophic adjustment" between the initial disturbed wind field and the undisturbed wind field. Thereafter, the error growth was exponential for the next seven days with a doubling time of about 2.5 days. Smagorinsky concluded that the deterministic limit of predictability for synoptic scale disturbances is about three weeks; however, the current practical limit is about one week. He also noted that short-wave predictability decays most rapidly.

The cause of the exponential growth of small errors in these studies was attributed to baroclinic instability [Charney, et al (1966)]. However, Lorenz (1972) suggested that the error growth might be due to barotropic instability of wave disturbances. In particular he investigated the stability of a finite amplitude unbounded Rossby wave. He found instability when the waves are sufficiently strong or the wave number is sufficiently high, and the growth rates are comparable to the growth rates obtained from predictability studies. Lilly (1973) emphasized the importance of barotropic instability in a predictability study which employed a 2-dimensional model. Recently, Lorenz (1973b) has concluded that baroclinic instability is the most important cause of lack of predictability in the atmosphere.

Lorenz (1969c) utilized approach b to obtain the rate of separation of two fields which were initially similar. Five years of twice-daily height values of the 200-, 500-, and 850-mb surfaces on a grid of 1003 points were obtained. A weighted root-mean-square height difference is used as a measure of the difference between two states, or the "error." For each pair of states occurring within one month of the same time of year, but in different years, the error was computed. Numerous mediocre analogues were found but there were no really good ones. The smallest errors had a doubling time of about eight days. Since larger errors grow less rapidly this is probably an overestimate of the doubling time. Extrapolation with the aid of a quadratic hypothesis indicates that very small errors double in about 2.5 days. This compares very well with the numerical results reported by Smagorinsky (1969).

In a more recent paper Lorenz (1973a) has used the same data set to investigate the range of predictability. States of the atmosphere separated by 12 days or less are found on the average to resemble each other more closely than randomly selected states, even after an adjustment for the seasonal trend has been made. Higher correlations were obtained with a form of damped persistence. These results demonstrate the existence of partial predictability of instantaneous weather patterns at least 12 days in advance.

Lorenz (1969b) followed approach c with a statistical treatment of the 2-dimensional vorticity equation. He derived an equation for the "error energy" which is obtained from the difference between two solutions. Thompson (1957) considered similar equations. The linearized equations are written in spectral form and ensemble averages are taken. A further assumption is included to close the set of predictive equations. The equations require the specification of a mean energy spectrum, and Lorenz used the -5/3 law for the smaller scales. The equations were integrated numerically from an initial error distribution; the error energy of each wave number was not allowed to exceed the mean atmospheric energy for that wave number. The numerical solutions show a rapid growth of the error for the very small scales and a very slow growth for large scales. Lorenz's solutions show that the range of predictability for cumulus scales is almost an hour, while the synoptic-scale motions can be predicted a few days ahead. Predictability for the largest scale disappears after about 17 days. In particular if the initial error is confined to the smallest scales the error in those scales rapidly reaches the maximum error. Then growth occurs in the next larger scales until the error is propagated to the largest scale. Lorenz points out that if the error in the smallest scale is reduced then the range of predictability of the largest scale features will only be increased by a time interval which is less than the range of predictability of the small scales where the error was reduced. Although these results are dependent on the closure assumption and the energy spectrum which is used the study shows clearly the importance of the nonlinear propagation of errors between different scales of motion.

Robinson (1967, 1971) assumed that the dynamic equations do not allow one to predict motions of a given scale over periods longer than the fluid elements of this scale maintain their identity against turbulent diffusion by smaller scale motions. Then based upon the dissipation characteristics of the atmosphere, he derived predictability ranges of a few days for synoptic-scale motions and about one hour for cumulus scale motions, which are roughly the same as obtained by Lorenz (1969b).

Leith (1971) and Leith and Kraichman (1972) have also used homogeneous isotropic turbulence models to study atmospheric predictability. Leith (1971) considered two dimensions and closed his equations with the eddy-damped Markovian approximation. Leith and Kraichman (1972) considered both two and three dimensions and they used Kraichman's (1971) closure method which is based on the test-field model. The solutions in three dimensions showed that even where there is a strong energy cascade there is also an upscale propagation of error. The two-dimensional solutions used a "-3" power energy spectrum for the smaller scales. Charney (1971, 1973) showed that quasi-geostrophic three-dimensional flow should have a "-3" power energy spectrum under proper conditions. Leith and Kraichman (1972) found that the two-dimensional flow was more predictable than three-dimensional flow. They concluded that an initial state determined with a horizontal resolution feasible with a satellite-based observing system would result in significant predictability of large scale motion for more than one week. They also point out that the test-field model probably underestimates rather than overestimates predictability times.

In summary, the studies of predictability suggest that specific weather patterns and events are predictable, or partially so, only for a period of at most several weeks; however, the possibility of predicting general trends of perhaps temperature and precipitation above or below normal for longer periods is not precluded as yet.

12. Stochastic Dynamic Prediction

Recent studies in stochastic dynamic prediction are closely related to atmospheric predictability. Epstein (1969) developed a method for including the effect of random errors in initial conditions in a forecast model. The forecast equations which he treated were of the spectral type with quadratic nonlinearities. He first took an ensemble average of the basic equations which leads to predictive equations for the ensemble average or the expected value of each dependent variable. These equations also include the variances and covariances between the dependent variables. Equations for the time change of the latter quantities which are second moments are obtained by multiplying each equation by each of the variables and carrying out the ensemble averages. These equations, however, contain the third moments. Equations for the time change of the third moments are obtained in the same way and they contain the fourth moments. This is merely the closure problem of classical turbulence theory.

Epstein closed his equations by neglecting the third moments about the instantaneous mean. He integrated the resulting equations for the Lorenz (1960) three component system. Initially he specified the expected value of each variable as well as its variance due to possible observational errors. The initial covariances were set equal to zero. A deterministic forecast was also performed with original prediction equations. Numerical integrations showed that the deterministic solution eventually diverges from the stochastic prediction of the expected value. The latter should be a better forecast in the statistical sense. The predicted variances grow in time which gives the influence of the initial error. Also a large number of deterministic forecasts were made with slightly different initial conditions. These numerical solutions were then averaged using a Monte Carlo method. The difference between the Monte Carlo average and the approximate stochastic solution is a measure of the error which is caused by the closure assumption. In some of the solutions these quantities remained close while in others they began to depart after a few days.

Fleming (1971a) has continued this work with a two-level quasi-geostrophic spectral model similar to the one used by Lorenz (1965). For interpretation he divided the energy into a "certain" energy and "uncertain" energy. He also reconsidered Epstein's closure assumption which neglected the third order moments. Fleming tested the quasi-normal approximation which computes the fourth order moments in the third order equations in terms of the second order moments. This requires the numerical solution of the third order equations in addition to the others. He found that this approximation was better up to a certain time, after which it became unstable. He then considered a modified form in which a linear damping term was added to the third order equations. This eddy-damped quasi-normal approximation was stable and gave excellent results in most cases.

Fleming (1971b) used the eddy-damped quasi-normal approximation in a stochastic formulation of the barotropic model. He used two wave numbers in latitude and 15 in longitude in a study of predictability. The predictability times obtained are similar to those obtained by Lorenz (1969b). Predictability studies using the baroclinic model of the earlier paper showed that predictability is increased when heating and friction are present. Fleming (1972) considered the effect of random variations in the thermal forcing and the effect of errors resulting from the improper treatment of the smaller scales.

Leith (1973) has concluded that the stochastic dynamic method cannot be used for the larger numerical models because of the very excessive computer time requirements. He suggests instead the use of the Monte Carlo procedure which involves a collection of deterministic forecasts. The procedure was evaluated with a simple two-dimensional turbulence model of large-scale atmospheric motions. A study of the dependence on sample size of mean square forecast error, both with and without a final linear regression correction,showed a considerable improvement with moderate sample sizes over conventional single forecasts without regression. These results suggest that a sample size of about 10 may be adequate for producing the error variance information needed for optimal data assimilation.

13. Spectral Methods

The spectral method expresses the spatial variation of the prediction fields in terms of a series of orthogonal functions. The coefficients in the series are now the forecast quantities rather than gridpoint values of the original dependent variables. We will illustrate the technique with the barotropic vorticity equation:

$$\frac{\partial}{\partial t} \nabla^2\psi + \mathbf{k} \times \nabla\psi \cdot \nabla(\nabla^2\psi) = 0 \tag{19}$$

where ψ is the stream function and the beta term has been neglected for simplicity. We expand ψ into the following series:

$$\psi(x_1,x_2,t) = \sum_{\alpha} \Psi_\alpha(t)\, Y_\alpha(x_1,x_2) \, . \tag{20}$$

The functions Y_α are orthogonal and normalized so that

$$\int Y_\beta^* \, Y_\alpha \, dS = \delta_{\alpha\beta} \, , \tag{21}$$

where Y_β^* is the complex conjugate of Y_β and

$$\delta_{\alpha\beta} = \begin{cases} 0 & \alpha \neq \beta \\ 1 & \alpha = \beta \end{cases} \, .$$

The integration in (21) is carried out over the entire forecast region. The functions Y_α satisfy the equation

$$\nabla^2 Y_\alpha + K_\alpha Y_\alpha = 0 \, , \tag{22}$$

where the eigenvalues K_α are positive and increase for the smaller scale eigenfunctions. Substitute (20) into the forecast equation (19), multiply by Y_γ^*, integrate over the entire region and use the relation (21) which gives

$$- K_\gamma \frac{d\Psi_\gamma}{dt} + \sum_\alpha \sum_\beta L_{\gamma,\alpha,\beta} \Psi_\alpha \Psi_\beta = 0 . \tag{23}$$

the interaction coefficients, which can be computed once and for all, are given by

$$L_{\gamma,\alpha,\beta} = - K_\beta \int Y_\gamma^* \mathbb{k} \cdot \nabla Y_\alpha \times \nabla Y_\beta \, dS . \tag{24}$$

Equation (23) represents an infinite number of ordinary differential equations when all appropriate values of γ are considered. In practice the series given by (20) is truncated in such a way that the desired meteorological features are properly described. The equations for the remaining coefficients can then be integrated in time numerically.

Lorenz (1960) treated equation (19) with this procedure in cartesian coordinates. In that case the eigenfunctions are products of sines and cosines and the interaction coefficients take on a particularly simple form. He noted that when the series is truncated total energy will still be conserved if the excluded coefficients are set to zero in the interaction sums. He also demonstrated the usefulness of very low order systems with his study of a 3-component set.

Silberman (1954) first applied this technique in spherical coordinates although he kept the zonal mean flow fixed in time. Platzman (1960) and Baer and Platzman (1961) performed a complete treatment of the barotropic vorticity equation in spherical coordinates and also carried out some experiments. In spherical coordinates the eigenfunctions are spherical harmonics which are composed of sines and cosines of the longitude and Legendre polynomials of the sine of the latitude. With these functions the interaction coefficients are more complicated and they have more non-zero elements than in cartesian coordinates.

The spectral method has several advantages over the gridpoint method. First the spectral method computes spatial derivatives exactly so that the phase error which occurs with the finite difference method is eliminated. Also the aliasing that occurs with finite differences is excluded and as a result it is easy to conserve certain quantities which are conserved in the continuous equations. Poisson equations are easily solved without relaxation or other inversion techniques because of relation (22). Another advantage is the treatment of global motions without the presence of singularities.

The most important disadvantage of the spectral method is that it requires much more computer time than the gridpoint method, if there are very many degrees of freedom. This can be seen from equation (23) which shows that for each degree of freedom many products must be computed in the nonlinear term. With the gridpoint method, for each degree of freedom, there are only a few products involving quantities at surrounding grid-points. The spectral method is generally more complicated to apply and it is not convenient for complicated boundaries. The spectral method also suffers when values must be evaluated locally such as in determining condensation criteria. We shall see that some of these difficulties have been alleviated with recent developments.

Robert (1966, 1968) modified the spectral technique in two studies with the primitive equations on the sphere. In place of the spherical harmonic functions he used the following function of sines and cosines:

$$G_n^m(\lambda,\varphi) = e^{im\lambda} \cos^{|m|}\varphi \sin^n\varphi \,. \qquad (25)$$

These functions are not orthogonal, but the spherical harmonic functions can be written as a sum as of the functions given by (25). The nonlinear terms are easily computed with these functions, and the number of elements involved is much less than with the spherical harmonics. When the orthogonality relations are required the spherical harmonics can be formed in terms of these functions. Using this simplified procedure Robert carried a number of spherical integrations with a relatively small number of terms.

A very important advance for the spectral method was the development of the fast Fourier transform by Cooley and Tukey (1965). Their technique allows Fourier transformation of a field with N degrees of freedom with order $N \log_2 N$ operations, while the direct method requires order N^2 operations. This allows rapid calculation of local fields by summing the series with the fast Fourier transformation, such as might be required for condensation tests.

Orszag (1969, 1970) has used the fast Fourier technique to save computation time in the evaluation of the nonlinear terms. He carries out all differentiation of quantities while they are represented by the series, but products of fields are performed with the fields at gridpoints. This requires an inverse Fourier transform to obtain data at gridpoints and a regular Fourier transformation to return to spectral form. The fast Fourier transformation can be used in both of these operations with a great time savings. When this procedure can be used it greatly reduces prediction time, for large numbers of variables, because the sum in (23) is replaced by a much faster operation.

Merilees (1973a) has developed an algorithm for computing the sum of a series of spherical harmonics. The algorithm is approximately 10-20 times faster than the standard method, but it suffers a precision problem and it breaks down when the resolution is too high. When this method is used, following Orszag, for rapid computation of the nonlinear terms, the prediction time for a global spectral model can be greatly reduced.

Bourke (1972) formulated a global spectral model based upon the one-layer shallow water equations. He used the vorticity and divergence equations instead of the 2 components of the equation of motion. The technique for evaluating the nonlinear terms which was developed by Orszag (1970) and Eliasen, et al (1970) is employed. The computational efficiency of the model was found to be far superior to that of an equivalent model based on the interaction coefficients. This model, in integrations of 116 days, satisfied the principles of conservation of energy, angular momentum and square potential vorticity to a high degree.

Daley (1973) used Bourke's (1972) model to examine the possibility of using different spatial resolution for different dependent variables. He suggested that for the smaller scales it would be sufficient to predict only the wind field because for those scales the pressure field adjusts to the wind field. In a test the smaller scales were predicted with a filtered model and the larger scales with the full primitive equations. He found that this combined system gave better forecasts than a primitive equation system with intermediate resolution. The application of this procedure to baroclinic equations may be more difficult.

Recent developments have made the spectral models much more competitive with gridpoint models with respect to computational efficiency. For the same resolution the spectral models can be expected to give better forecasts, and the forecasts may be better even with somewhat poorer resolution. A series of tests are needed to determine whether or not better forecasts can be made with the spectral method with the same computational effort.

Orszag (1972) and Merilees (1973b) have proposed a forecast technique which is a combination of the finite difference method and spectral method. This technique which is known as the pseudospectral approximation employs finite differences except in the computation of spatial derivatives. Before the derivatives are computed, the gridpoint data is fast Fourier analyzed. Continuous derivatives are then computed and the series is summed with the fast Fourier transform. This procedure is used for both longitudinal and latitudinal derivatives. The latitudinal derivatives in some cases require a sign change at the poles. Merilees (1973b) found excellent results in a test prediction of Haurwitz waves with the shallow water equations.

14. Finite Element Method

The extensive success of the finite element method in solid mechanics has attracted the interest of investigators in other fields. Salinas (1973) has given a brief review of the method which proceeds as follows: At the start, an approximate solution in the form of a linear combination of specified (basis) functions is assumed. The coefficients (multipliers) of the basis functions are to be determined to yield the best approximate solutions. This is accomplished by minimizing a measure of the error (called the residual function) associated with the assumed solution. Several techniques are available for minimizing the residuals. Normally the basis functions are defined in such a way that they have a simple variation (quadratic, cubic, etc.) over an element of area (piecewise polynomials), outside of which they are zero. The "elements" can have a variety of shapes. The finite element method is a generalization of the method of weighted residuals.

Newton (1973) has successfully applied the technique to gravity waves radiating out from an oscillating ship. Gallagher and Chan (1973) have treated the steady state circulation in Lake Ontario. Triangular and otherwise shaped elements were used to accurately fit the lake's coastline. Wang, et al (1972) applied the method to the one-dimensional shallow water equations. They obtained better results for both the linear and nonlinear cases than with the usual finite difference methods. Price and MacPherson (1973) have applied the technique to the two-level general circulation model developed by Mintz and Arakawa (Langlois and Kwok, 1969). They arranged the elements in such a way that the area of each element is nearly constant over the entire globe. They also provided for a subregion in which the elements telescoped to a smaller size. Predictions with this model gave encouraging results.

For meteorology, the principal advantage of the method as shown by Price and MacPherson (1973), is the possibility of easily changing the element size and shape. This would be useful in situations where meshed models are now used and also on the sphere where the elements could be kept at a fixed scale.

15. Summary

Research on initialization has intensified in the last few years. Most operational forecast models employ the primitive equations which are very sensitive to the initial balance between the wind and the pressure fields. Short range prediction of precipitation is also very sensitive to the initial divergence field. The development of global models requires proper initialization in the tropics which may differ from middle latitudes. The most promising initialization techniques include the variational formulations and the averaging of backward and forward forecasts about the initial state. The assimilation of nonsynoptic data into a forecast model has received considerable attention. This has been motivated by availability of nearly continuous satellite data. The main problem here involves controlling the imbalance introduced into the forecast by the localized new data.

Several groups have developed limited area models which are designed to give better forecasts in specific regions. These models use boundary conditions from hemispheric or global models and in some cases the limited area forecast is allowed to affect the exterior region. Isentropic models have been developed and applied to smaller scale synoptic features.

Operational predictions of pressure and wind have shown continuing improvement as a consequence of progressively better numerical models. However, the concomitant improvement in forecasting precipitation and temperature has not kept the same pace. It is expected that forecasts of weather elements will improve as the limited area models produce better descriptions of mesoscale systems.

Studies of predictability suggest that specific weather patterns and events are predictable, or partially so, only for at most several weeks. The possibility of predicting general trends above or below normal for longer periods is not as yet precluded. Perhaps the consideration of atmosphere-ocean interaction will lead to longer range prediction of weather trends.

The stochastic dynamical prediction procedure is closely related to studies of atmospheric predictability. The procedure uses the initial uncertainty in the initial state to predict the uncertainty in the forecast at later times as well as the expected value. This method cannot be used for operational forecasts in the near future because of the large computer time requirements. However, an indication of the growth of uncertainty can be obtained by examining a relatively small number of deterministic forecasts with slightly different initial conditions.

The development of the semi-implicit method is a major advance in finite differencing. The method treats the gravity wave terms implicitly and the advection terms explicitly. This permits the use of a longer time step with the additional requirement that an elliptic equation be solved at each time step. The net effect of this is a reduction in computer time by a factor of at least 2. This has lead to the development of new methods for solving elliptic equations which are faster and more accurate than the traditional relaxation methods.

Present operational forecasts often underpredict the movement of synoptic disturbances and this error has been attributed to space truncation error in the finite difference equations. Several groups are experimenting with fourth order space differencing in order to reduce this error. Also the pseudospectral method has been put forward as a method to eliminate space truncation effects.

New interest has developed in the use of the spectral method since the introduction of the fast Fourier transform. The use of this transform greatly speeds up computation time with the spectral method. Further tests are required to determine whether the spectral method will give better forecasts than the finite difference method for the same amount of computer time.

The finite element method of solid mechanics has been recently introduced into meteorology. This method has the advantage of a very flexible element size, but it has not been widely tested on meteorological problems.

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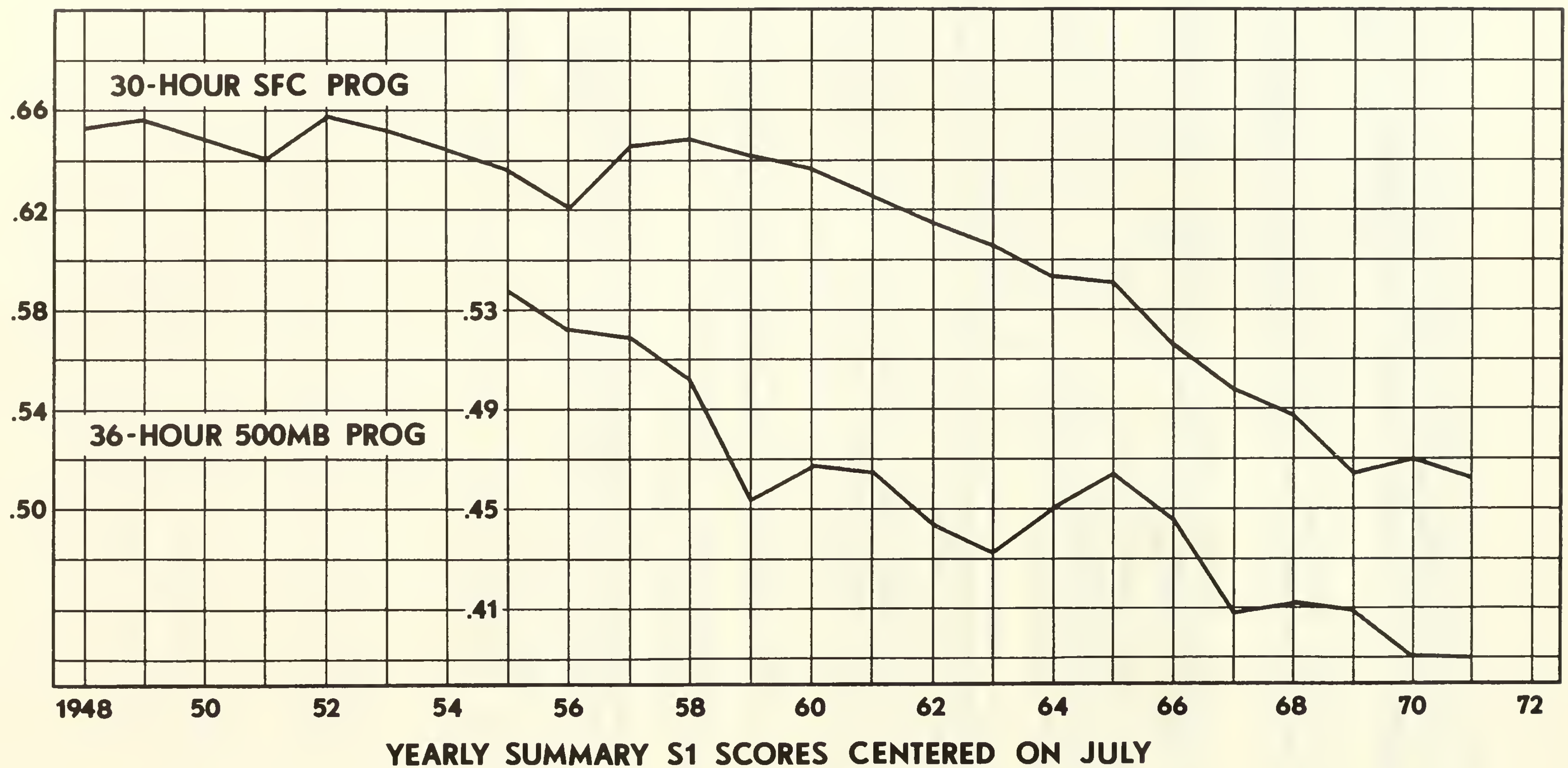

Figure 1. (after Shuman (1972)) - Annual average S1 scores (Teweles and Wobus (1951)) for 30 hr sea level (upper curve) and 36 hr 500 mb (lower curve) forecasts. The S1 score is roughly a measure of normalized RMS vector error of pressure gradient. The area of verification for both levels covers North America. The two curves are plotted on the different scales shown. The scale for sea level is the one labelled from .50 to .66, the scale for 500 mb from .39 to .54. To calibrate the scores in terms of practical skill, a sea-level forecast with a score of .30 is virtually perfect, one with a score of .80 is worthless. For 500 mb, .20 represents a virtually perfect forecast, .70 worthless.

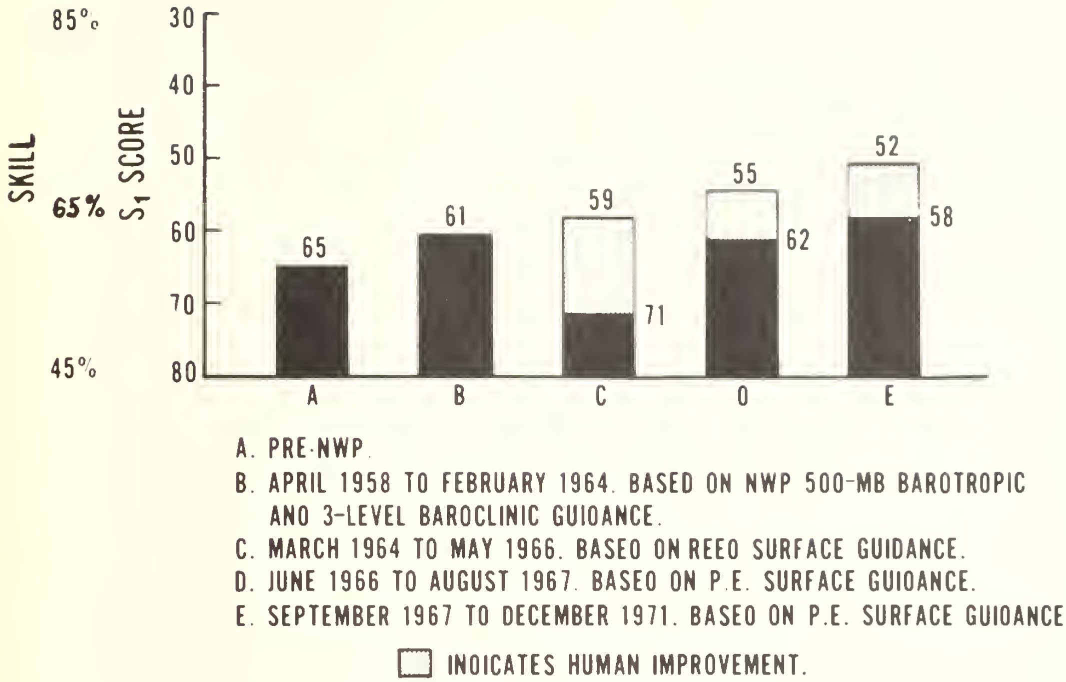

Figure 2.--NMC average 30-hour surface S_1 scores

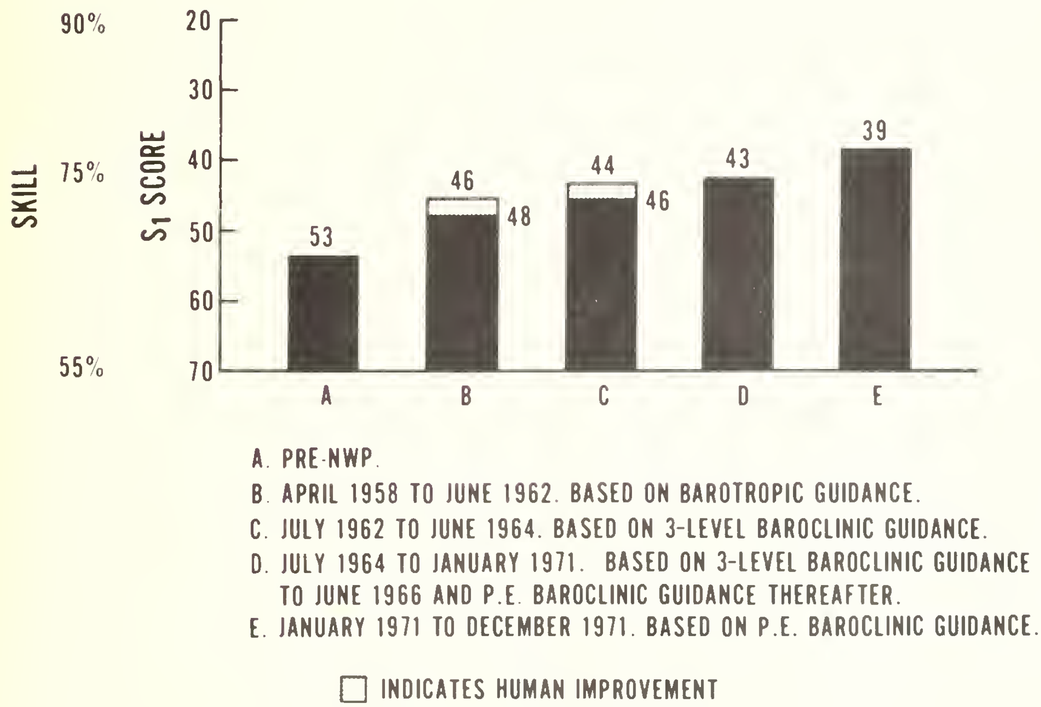

Figure 3.--NMC average 36-hour 500-mb S_1 scores

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